Why Classical Space is Not a Limit of Minkowski Space

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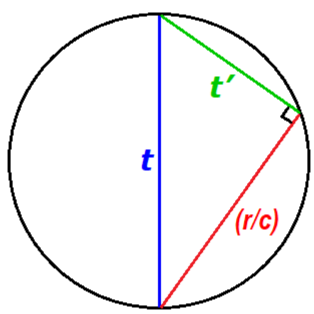
7:49 AM, September 23, 2020 (Backreaction Post Time)

Includes new figures and minor edits. Original text-only Backreaction version is at:
https://backreaction.blogspot.com/2020/09/path-dependence-and-tipping-points.html?showComment=1600861785891#c6021323565412361310

Having previously mentioned [1] Minkowski's 1908 thought experiment about altering the speed of light, [2] I should also point this out: His conclusion from that argument is simply wrong.

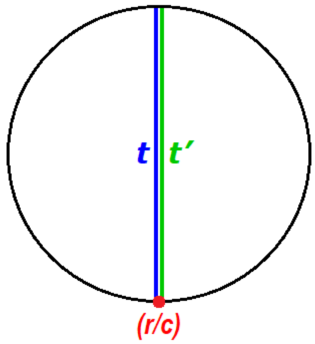
What Minkowski claimed was that if you increase the speed of light to infinity, the result is a symmetry group that is equivalent to classical, non-relativistic space. Alas, what you actually get is a mathematical singularity in which motion of any type is impossible — a boring, point-like space in which nothing can happen.

The easiest way to show why this is true is to use the Euclidean special relativity right triangle $\langle t{:}\,(r/c), t' \rangle$ that I described in an earlier Backreaction comment:

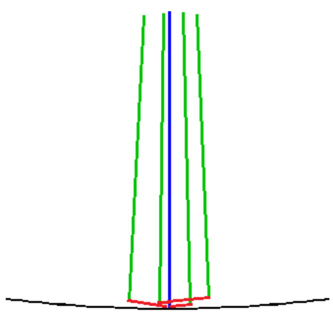


The vertical hypotenuse $\boldsymbol{t}$ is rest-frame (observer) elapsed clock time, the lower $(\boldsymbol{r}/\boldsymbol{c})$ leg is distance traveled by the moving (observed) object, and the upper $\boldsymbol{t}'$ leg is clock time elapsed by a clock on the moving (observed) object.

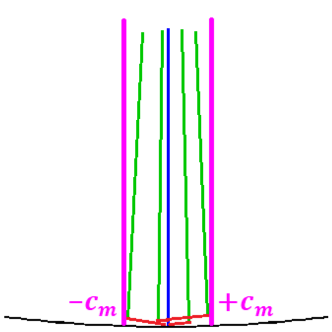
The problem is not complicated: If you set $c = \infty$, then $(\boldsymbol{r}/\boldsymbol{c}) = \mathbf{0}$ and all motion — all concept of spatial distance — disappears, leaving a point-like space moving only through time:



What Minkowski was attempting to get at was that the larger the ratio of c is to the typical velocities of physical objects, the more "Euclidean-like" the spatial subset of spacetime will appear. This case is best represented not by a V-shaped hyperbola (Minkowski's infinite c), but by setting c to a finite velocity and instead insisting that the speeds of material objects remain very small in comparison to c. This situation is always just a convenient subset and never a true mathematical limit — that is, it is the difference between *approaching* an infinity versus actually *reaching* that infinity. But in order to obtain a space of any size at all, one must remain in the domain of "approaching" rather than "completing".



The figure that best represents actual classical space is the group of right triangles for which the distance-traveled sides $(\boldsymbol{r}/\boldsymbol{c})$ at the bottom of Euclidean spacetime are all "very short", causing their remaining $\boldsymbol{t}'$ sides to be very close to $\boldsymbol{t}$ — that is, the right triangles for which there is no appreciable time dilation, and making them all approximately (but never exactly) orthogonal to time $\boldsymbol{t}$. This is of course the situation we humans live in most of the time, and it enables us to approximate xyz as simple Euclidean with fully orthogonal time. We perceive space as the set of very short $(\boldsymbol{r}/\boldsymbol{c})$ legs at the bottom of a set of very skinny right triangles. Because this set of very short triangle legs is almost perpendicular to time, staying within it allows us to get away with using this sloppy approximation of the space and time relationship.



There remains a path by which a *completed* integral could define classical space, but to be mathematically self-consistent this path would require that material objects — objects with non-zero rest mass — have a *different* absolute speed limit from that of light. This second speed limit for massive objects would need to be very small in comparison to the velocity of massless c photons, but would similarly require infinite energy for material objects to reach it fully, just is in real special relativity. Notably, even this result would remain an approximation of classical space, since it would only be *approximately* flat in the classical sense. But more importantly, such a two-limit construct would literally have nothing to do with what is actually *observed* in our universe, which is that massive and massless objects both respond (albeit with different energy profiles) to a *single* speed limit, c.

The bottom line is that rather than being a simple limit to the Poincaré symmetries as Minkowski speculated, classical space can never be accurately modeled as anything more than an illusion enabled by c being much larger than human velocities. That is all classical space really is and ever can be: An approximation.

References

[1] T. Bollinger, "Bollinger reply to JimV," 2020-09-22.13:29. [Online]. Available: https://backreaction.blogspot.com/2020/09/path-dependence-and-tipping-points.html?showComment=1600795787571#c1441430986227904723. [Accessed 2020-09-26].

[2] H. Minkowski, "Space and Time," *80th Assembly of German Natural Scientists and Physicians,* 1908.